

# Function

▶ Start

# Function

▶ Start

A function  $f$

# Function

▶ Start

A function  $f$  from X

# Function

▶ Start

A function  $f$  from X to Y

# Function

▶ Start

A function  $f$  from X to Y is a subset

# Function

▶ Start

A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$

# Function

▶ Start

A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$   
subject

# Function

▶ Start

A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$  subject to the following condition:

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A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$  subject to the following condition:

Every element

# Function

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A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$   
subject to the following condition:

Every element of  $X$

# Function

▶ Start

A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$  subject to the following condition:

Every element of  $X$  is the first component

# Function

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A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$  subject to the following condition:

Every element of  $X$  is the first component of one and only one

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A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$  subject to the following condition:

Every element of  $X$  is the first component of one and only one ordered pair

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A function  $f$  from  $X$  to  $Y$  is a subset of the Cartesian product  $X \times Y$  subject to the following condition:

Every element of  $X$  is the first component of one and only one ordered pair in the subset.

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In other words,

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In other words, for every  $x$

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$X \times Y$

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$$X \times Y =$$

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$$X \times Y = \{(x, y)$$

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$X \times Y = \{(x, y) | x \in X, y \in Y\}$  : The Cartesian product

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$$\left\{ \begin{array}{l} f \subset \\ \quad \end{array} \right.$$

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$f$

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$f :$

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$$f : X$$

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 $f$  is a function from  $X$  to  $Y$ .

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$$f : X \rightarrow$$

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$$f : X \rightarrow Y,$$

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$$f : X \rightarrow Y, X$$

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f}$$

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f$$

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f :$$

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x$$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow$$

# Function

▶ Start

$X \times Y = \{(x, y) | x \in X, y \in Y\}$ : The Cartesian product  $X \times Y$   
 $f$  is a function from  $X$  to  $Y$ .

$$\begin{cases} f \subset X \times Y \\ \forall x \in X, \exists ! y \in Y \text{ s.t. } (x, y) \in f \end{cases} \begin{cases} \forall x \in X, \exists y \in Y \text{ s.t. } (x, y) \in f \\ (x, y_1) \in f \text{ and } (x, y_2) \in f \Rightarrow y_1 = y_2 \end{cases}$$

$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y,$$

# Function

▶ Start

$X \times Y = \{(x, y) | x \in X, y \in Y\}$ : The Cartesian product  $X \times Y$   
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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x$$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f}$$

# Function

▶ Start

$X \times Y = \{(x, y) | x \in X, y \in Y\}$ : The Cartesian product  $X \times Y$   
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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y,$$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y$$

# Function

▶ Start

$X \times Y = \{(x, y) | x \in X, y \in Y\}$ : The Cartesian product  $X \times Y$   
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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y =$$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f$$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f($$

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$$f(x)$$

# Function

▶ Start

$X \times Y = \{(x, y) | x \in X, y \in Y\}$ : The Cartesian product  $X \times Y$   
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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value

# Function

▶ Start

$X \times Y = \{(x, y) | x \in X, y \in Y\}$ : The Cartesian product  $X \times Y$   
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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$

# Function

▶ Start

$X \times Y = \{(x, y) | x \in X, y \in Y\}$ : The Cartesian product  $X \times Y$   
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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

# Function

▶ Start

$X \times Y = \{(x, y) | x \in X, y \in Y\}$ : The Cartesian product  $X \times Y$   
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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

$X$  Domain

$Y$  Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$

# Function

▶ Start

$X \times Y = \{(x, y) | x \in X, y \in Y\}$ : The Cartesian product  $X \times Y$   
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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

# Function

▶ Start

$X \times Y = \{(x, y) | x \in X, y \in Y\}$ : The Cartesian product  $X \times Y$   
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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$f(X)$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(X) =$$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(X) = \{$$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(X) = \{y$$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(X) = \{y |$$

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(X) = \{y | \exists x$$

# Function

▶ Start

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(X) = \{y | \exists x \in$$

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(X) = \{y | \exists x \in X$$

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

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$y$  The dependent variable

$f(X) = \{y | \exists x \in X \text{ s.t. }$

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

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$x$  The independent variable

$y$  The dependent variable

$$f(X) = \{y | \exists x \in X \text{ s.t. } y = f(x)\}$$

# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

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$y$  The dependent variable

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# Function

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$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(X) = \{y | \exists x \in X \text{ s.t. } y = f(x)\} =$$

# Function

▶ Start

$X \times Y = \{(x, y) | x \in X, y \in Y\}$ : The Cartesian product  $X \times Y$   
 $f$  is a function from  $X$  to  $Y$ .

$$\begin{cases} f \subset X \times Y \\ \forall x \in X, \exists ! y \in Y \text{ s.t. } (x, y) \in f \end{cases} \begin{cases} \forall x \in X, \exists y \in Y \text{ s.t. } (x, y) \in f \\ (x, y_1) \in f \text{ and } (x, y_2) \in f \Rightarrow y_1 = y_2 \end{cases}$$

$$f : X \rightarrow Y, X \xrightarrow{f} Y$$

X Domain

Y Codomain

$$f : x \longrightarrow y, x \xrightarrow{f} y, y = f(x)$$

$f(x)$  The value of a function  $f$  at  $x$

$x$  The independent variable

$y$  The dependent variable

$$f(X) = \{y | \exists x \in X \text{ s.t. } y = f(x)\} = \{$$

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