

The limit of a constant times a function is the constant times the limit of the function.

함수의 상수배의 극한은 함수의 극한의 상수배이다.
(The limit of a constant times a function is the constant times the limit of the function.)

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▶ Start

▶ End

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Theorem

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▶ Start

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Theorem

$c : \text{constant}$

The limit of a constant times a function is the constant times the limit of the function.

▶ Start

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Theorem

$$c : \text{constant}, \lim_{x \rightarrow a} f(x) = L$$

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Theorem

$c : \text{constant}, \lim_{x \rightarrow a} f(x) = L$

$$\lim_{x \rightarrow a} cf(x)$$

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$$\lim_{x \rightarrow a} cf(x) = cL$$

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Proof.



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$\epsilon > 0$



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$$|cf(x) - cL|$$



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$$|cf(x) - cL| = |c| \cdot |f(x) - L|$$



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Github:

<https://min7014.github.io/math20231128002.html>

Click or paste URL into the URL search bar,
and you can see a picture moving.